

Project 2 - Working with Matrices

In this project you will learn how to use Maple to perform algebraic operations with matrices and will practice some of the properties associated with invertible matrices.

Basic Maple Key Strokes

$2*3$ for multiplying two numbers

$2*A$ for multiplying a number and a matrix

$A.B$ for multiplying matrices of the appropriate size

A^2 for raising a matrix to a positive integer power

Part A: Algebra with Matrices

Consider the examples below. Notice how the key strokes above do not look the same once typed.

```
> restart: with(LinearAlgebra):
```

```
> A := Matrix([[1, 2, 5], [0, -4, 7]]);
```

$$A := \begin{bmatrix} 1 & 2 & 5 \\ 0 & -4 & 7 \end{bmatrix} \quad (1)$$

```
> B := Matrix([[1, 0, -3], [4, 1, 5]]);
```

$$B := \begin{bmatrix} 1 & 0 & -3 \\ 4 & 1 & 5 \end{bmatrix} \quad (2)$$

```
> C := Matrix([<1, 2, 3>, <-1, 5, 2>, <0, 1, 1>, <-2, -3, 0>]);
```

$$C := \begin{bmatrix} 1 & -1 & 0 & -2 \\ 2 & 5 & 1 & -3 \\ 3 & 2 & 1 & 0 \end{bmatrix} \quad (3)$$

```
> A + B;
```

$$\begin{bmatrix} 2 & 2 & 2 \\ 4 & -3 & 12 \end{bmatrix} \quad (4)$$

```
> MatrixAdd(A, B);
```

$$\begin{bmatrix} 2 & 2 & 2 \\ 4 & -3 & 12 \end{bmatrix} \quad (5)$$

```
> A • C;
```

$$\begin{bmatrix} 20 & 19 & 7 & -8 \\ 13 & -6 & 3 & 12 \end{bmatrix} \quad (6)$$

$$\begin{aligned} &> \text{MatrixMatrixMultiply}(A, C); \\ &\quad \begin{bmatrix} 20 & 19 & 7 & -8 \\ 13 & -6 & 3 & 12 \end{bmatrix} \end{aligned} \tag{7}$$

$$\begin{aligned} &> 7 \cdot A; \\ &\quad \begin{bmatrix} 7 & 14 & 35 \\ 0 & -28 & 49 \end{bmatrix} \end{aligned} \tag{8}$$

$$\begin{aligned} &> 5 \cdot A - 4 \cdot B; \\ &\quad \begin{bmatrix} 1 & 10 & 37 \\ -16 & -24 & 15 \end{bmatrix} \end{aligned} \tag{9}$$

$$\begin{aligned} &> P := \text{Matrix}([\langle 1, 2 \rangle, \langle -3, 7 \rangle]); \\ &\quad P := \begin{bmatrix} 1 & -3 \\ 2 & 7 \end{bmatrix} \end{aligned} \tag{10}$$

$$\begin{aligned} &> P \cdot P \cdot P; \\ &\quad \begin{bmatrix} -53 & -153 \\ 102 & 253 \end{bmatrix} \end{aligned} \tag{11}$$

$$\begin{aligned} &> P^3; \\ &\quad \begin{bmatrix} -53 & -153 \\ 102 & 253 \end{bmatrix} \end{aligned} \tag{12}$$

Part B: Defining special matrices

The identity matrix

$$\begin{aligned} &> \text{restart} : \text{with}(\text{LinearAlgebra}) : \\ &> \text{IdentityMatrix}(3); \\ &\quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned} \tag{13}$$

$$\begin{aligned} &> \text{IdentityMatrix}(7); \end{aligned} \tag{14}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(14)

> IdentityMatrix(3, 5);

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

(15)

The zero matrix.

> ZeroMatrix(3);

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(16)

> ZeroMatrix(7);

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(17)

> ZeroMatrix(2, 3);

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(18)

Part C: Determining if a matrix is invertible or singular.

In this case, the matrices need to be square matrices. Here are examples of outputs indicating errors when working with non-square matrices.

```
[> restart : with(LinearAlgebra) :
```

```
> Matrix([[1, 2, 3], [4, 5, 0]]);
```

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 0 \end{bmatrix}$$

(19)

```
> Determinant(A);
```

Error, (in LinearAlgebra:-Determinant) invalid input: LinearAlgebra:-Determinant expects its 1st argument, A, to be of type Matrix() but received A

```
[>
```

```
> MatrixInverse(A);
```

Error, (in MatrixInverse) MatrixInverse expects its 1st argument, M, to be of type {Matrix, [Matrix(square)], [Vector, Matrix(square)], [Matrix(square), Matrix(square), Matrix(square)]} but received A

Let's now continue working with square matrices.

An example of an invertible matrix.

```
> restart : with(LinearAlgebra) :
```

```
> A := Matrix([[1, 2], [3, 4]]);
```

$$A := \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

(20)

```
> ReducedRowEchelonForm(A);
```

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(21)

```
> Determinant(A);
```

$$-2$$

(22)

```
> MatrixInverse(A);
```

$$\begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$$

(23)

```
> A-1;
```

(24)

$$\left[\begin{array}{cc} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{array} \right] \quad (24)$$

An example of a singular matrix.

$$\begin{aligned} &> B := \text{Matrix}([\langle 1, 0, -1 \rangle, \langle 0, 1, 1 \rangle, \langle 2, 1, -1 \rangle]); \\ & \quad B := \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ -1 & 1 & -1 \end{bmatrix} \end{aligned} \quad (25)$$

$$\begin{aligned} &> \text{ReducedRowEchelonForm}(B); \\ & \quad \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{aligned} \quad (26)$$

$$\begin{aligned} &> \text{Determinant}(B); \\ & \quad 0 \end{aligned} \quad (27)$$

Project 2 – Working with Matrices

Instructions:

- Submit your work by the deadline. Projects can only be submitted for grading using Canvas. No late projects will be accepted.
- Don't forget to remove output before submitting your work for grading.
- Name your file correctly (lastname_first name_project1)
example: castroconde _miriam_project1
- Check the top of your work to make sure you have included the required information.
- Only Maple commands learned in projects 1 and 2 can be used to solve the problems in this project.
- Insert text to explain your work and give answers.

For problems 1-7: $A = \begin{bmatrix} 1 & -2 & -1 \\ 2 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

1. Define a matrix P that can be added to A and compute $5A + P$ and $3A - 2P$ using Maple.
2. Define a matrix R for which RA is defined and compute the product $(2R)A$ using Maple.
3. Define a matrix Q such that AQ is a square matrix. Use Maple to compute AQ and call the product M .
4. Find the reduced row echelon form of M defined in problem 3 to determine if it is invertible. Explain your answer.
5. Define a matrix N such that NA is a square matrix. Use Maple to compute NA and call the product Z .
6. Compute the determinant of Z defined in problem 5, and use it to determine if the matrix is invertible or singular. Explain your answer.
7. Let T be the matrix transformation defined by A . Use the reduced row echelon form of A to determine if the transformation is one-to-one and onto. Justify your answer.

For problems 8 – 10: Consider the vectors below

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, v_2 = \begin{bmatrix} -1 \\ 0 \\ -2 \\ 0 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, v_4 = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 1 \end{bmatrix}, v_5 = \begin{bmatrix} 2 \\ 4 \\ 6 \\ 8 \end{bmatrix}$$

8. By looking at the vectors ONLY (don't do any work), give three reasons why they form a linearly dependent set. Explain your answers.
9. Show the vectors are linearly dependent by solving the appropriate vector equation. Explain your work.
10. Show the vectors are linearly dependent by working with the matrix having the vectors as columns. Explain your work.